# X-1. THEORETICAL ASPECTS OF 3-PORT JUNCTION CIRCULATORS


## H. Bosma (Invited)

*Philips Research Laboratories, Eindhoven, Netherlands*


Analysis of scattering matrices of lossless 3-port junctions with 3-fold rotational symmetry shows that circulators are obtained when the eigenvalues, which have unit magnitude, make angles of 120 degrees.[1] Zero reflection, zero insertion loss, and infinite isolation coincide at the same values of device parameters such as frequency, saturation magnetization, and size of ferrite.

When the junction is lossy, matters are more complicated. The eigenvalues no longer have unit magnitude. Let us agree that a 3-port junction is a circulator when, simultaneously, the reflection is zero and the isolation is infinite. For having a circulator it is then necessary that the eigenvalues not only make angles of 120 degrees, but also have equally large magnitudes. Hence, in the lossy case there is an additional condition. In many cases the two conditions cannot be satisfied simultaneously, resulting in asymmetry in device properties.

To understand this in more detail, let us next consider H-plane 3-port junctions. It is assumed that the electric field is purely in the axial direction and that, moreover, field components do not depend on the axial coordinate. Let the ideal, i.e. lossless, circulator be the unperturbed junction. Two types of perturbations can be introduced: (1) material losses, in terms of the imaginary parts of the electric permittivity, $i\epsilon''$, and of the elements of the permeability tensor, $i\mu''$ and $i\kappa''$; and (2) variations of frequency, $\delta\omega$, and bias field, $\delta\omega_H$ ($\omega_H = \gamma H$). The relative deviations of the eigenvalues from their "ideal" values, ($\delta\lambda_i/\lambda_i$), can be derived as follows:

$$\delta\lambda_i/\lambda_i = -\alpha_i + \beta_i, \qquad i = 0, +, -,$$

$$\alpha_i = \frac{\omega\zeta_0 k_0}{2F k_g}\int_V \left[\epsilon_0\epsilon''\,|E_i|^2 + \mu_0\mu''\,|H_i|^2 + i\mu_0\kappa''(H_{i\rho}H_{i\phi}^* - H_{i\rho}^* H_{i\phi})\right] d\tau$$

$$\beta_i = \left[\frac{\omega\zeta_0 k_0}{2F k_g}\int_V \left[\mu_0\omega_H\frac{\partial\mu}{\partial\omega_H}\,|H_i|^2 + i\mu_0\omega_H\frac{\partial\kappa}{\partial\omega_H}(H_{i\rho}H_{i\phi}^* - H_{i\rho}^* H_{i\phi})\right] d\tau\right]\frac{\partial\omega_H}{\omega_H} +$$

$$\left[\frac{\omega\zeta_0 k_0}{2F k_g}\int_V \left[\epsilon_0\epsilon\,|E_i|^2 + \mu_0\left(\mu + \omega\frac{\partial\mu}{\partial\omega}\right)|H_i|^2 + i\mu_0\left(\kappa + \omega\frac{\partial\kappa}{\partial\omega}\right)(H_{i\rho}H_{i\phi}^* - H_{i\rho}^* H_{i\phi})\right] d\tau\right]\frac{\partial\omega}{\omega}$$

The integrals must be taken over the volume of the center region. $k_0$ and $k_g$ are the propagation numbers of free space and the transmission lines, respectively, while $\zeta_0$ is the free space wave impedance. The quantity F is an integral of the normalized transverse field over the cross-section of the line. The subscript i = 0, +, -, refers to the three eigenvectors. $E_i$ and $H_i$ are the fields which are present in the center region, when the junction is excited by the $i^{th}$ normalized eigenvector.

It is readily seen that losses diminish the magnitudes of the eigenvalues. Variations of frequency and bias field rotate the eigenvalues in the complex plane. The latter can be used to readjust the circulator. The condition for zero reflection is given by:

$$\beta_+ - \beta_0 = \frac{1}{\sqrt{3}} ( 2 \alpha_- - \alpha_+ - \alpha_0 ) ,$$

$$\beta_- - \beta_0 = - \frac{1}{\sqrt{3}} ( 2 \alpha_+ - \alpha_- - \alpha_0 ) ,$$

The condition for infinitely large isolation is:

$$\beta_+ - \beta_0 = - \frac{1}{\sqrt{3}} ( 2 \alpha_- - \alpha_+ - \alpha_0 ) ,$$

$$\beta_- - \beta_0 = \frac{1}{\sqrt{3}} ( 2 \alpha_+ - \alpha_- - \alpha_0 ) .$$

In general, these two conditions are conflicting; they can be fulfilled simultaneously only if:

$$\alpha_+ = \alpha_- = \alpha_0$$

This means that, in order to have a circulator, the losses of the three eigenvector excitations must be equally large. Otherwise, zero reflection and infinite isolation will occur at different values of frequency and bias field. This has been confirmed by experiment.

Evaluation of the fields $E_i$ and $H_i$ is a fairly intricate problem. It has been solved only for some special configurations -- the waveguide circulator with central post[2,3] and the stripline circulator[4,5].

When the field in the junction is known, the above perturbation formulas can be used to evaluate such properties as bandwidth, tuning range, and temperature dependence. Making some simplifying assumptions, the bandwidth and the insertion loss of the stripline circulator can be derived. From the expressions so obtained, it can be shown that the circulator can be designed for optimum bandwidth and insertion loss. Moreover, requirements for the ferrite material can be formulated.

In the elementary form of the stripline circulator the ferrite discs and the center conductor have equally large diameters. In actual designs the center conductor is often taken smaller. Experiments have shown that in that configuration the boundary value problem of the electromagnetic field can be applied to the region covered by the center conductor. The outer ring of ferrite is inactive. Those sections of the input lines which extend between these rings perform as matching line sections. This results in an increased bandwidth. It is believed that the increase of bandwidth is largest if the sections are a quarter wavelength long. Let $R_f$ be the radius of the ferrite discs, and $R_c$ of the center conductor. We then have:[4]

$$R_c = \frac{1.84}{2\pi} \lambda \qquad \text{and} \qquad R_f - R_c = \lambda / 4$$

from which we obtain: $R_f / R_c = 1.85$ .
This has been confirmed experimentally by Simon.[6]

References.

1. B. A. Auld, IRE Trans. MTT-7, p 238, April 1959.
2. H. J. Butterweck, A.E.Ue. Vol. 17, p 163, April 1963.
3. J. B. Davies, IRE Trans. MTT-10, p 596, November 1962.
4. H. Bosma, IEEE Trans. MTT-12, p 61, January 1964.
5. J. B. Davies and P. Cohen, IEEE Trans. MTT-11, p 506, November 1963.
6. J. W. Simon, IEEE Trans. MTT-13, p 335, May 1965.